

Colegiul Național „Mircea cel Bătrân”, Râmnicu-Vâlcea
Concursul Interjudețean „Mathematica – Modus Vivendi”

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BAREM CLASA a VIII -a

- 1. a.** $t_a = \frac{a+1}{a-1} \sqrt{a^4 - 2a^2 + 1}$1 p
 $t_a = (a + 1)^2$1 p
 $a = 44$1 p
b. $(a - b)(a - 4b) = 0$2 p
 $a = 4b$1 p
 $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = 0, (3)$1 p

Total = 7 puncte

- 2. a.** $(a - b)^2 \geq 0, \forall a, b \geq 0$ 1 p
 Egalitatea are loc pentru $a=b$1 p
b. $2025! = (1 \cdot 2025) \cdot (2 \cdot 2024) \cdot \dots \cdot (1012 \cdot 1014) \cdot 1013$2 p
 $(1 \cdot 2025) \cdot (2 \cdot 2024) \cdot \dots \cdot (1012 \cdot 1014) \cdot 1013 < \left(\frac{1+2025}{2}\right)^2 \cdot \left(\frac{2+2024}{2}\right)^2 \cdot \dots \cdot \left(\frac{1012+1014}{2}\right)^2 \cdot 1013$1 p
 $2025! < 1013^{2+2+\dots+2} \cdot 1013$1 p
 $2025! < 1013^{2 \cdot 1012 + 1}$1 P

Total = 7 puncte

- 3. a.** $AP \perp MN$1 p
 $\Delta PAN \sim \Delta DMN \Rightarrow PA = \frac{24}{5}$1 p
 $SP = \frac{6\sqrt{241}}{5}$1 p
b. $\sphericalangle((SAP), (SAC)) = \sphericalangle PAC$ 1 p
 $QC \perp MN, \Delta MQC \sim \Delta MDN \Rightarrow QC = \frac{12}{5}$1 p
 $QT \perp PA, QC = TA$1 p
 $\cos \sphericalangle CAT = \frac{TA}{AC} = \frac{\sqrt{2}}{10}$1 p

Total = 7 puncte

- 4. a.** Construim $DE \equiv MB, D \in (EN)$1 p
 $\sphericalangle EAD = \sphericalangle MAB = \sphericalangle NAM = \alpha$1 p
 $\sphericalangle EAN = \sphericalangle NEA = 90^\circ - \alpha \Rightarrow \Delta NEA$ isoscel.....1 p
 $AN = ED + DN = MB + DN$1 p
b. $\Delta AOB \equiv \Delta C'OP \Rightarrow C'P = 6 \text{ cm}$ 1 p
 Fie $PE \perp DC$ și $EF \perp AC \Rightarrow EF = 3\sqrt{2}$1 p
 $d(P, AC) = PF = 3\sqrt{6}$1 p

Total = 7 puncte