CONCURSUL NAȚIONAL DE MATEMATICĂ
„TEHNICI MATEMATICE"-editia a XIX-a
Etapa națională 23.03.2024
Clasa a X -a Matematică *M_șt-nat*

Barem de corectare

Subiectul I (30p)

a) $D = [-5, \infty)$ 1p

Notăm $a = \sqrt{x+5}\ ; a \geq 0$

$b = \sqrt[3]{12-x}\ ; b \in \mathbb{R}$

$\begin{cases} a+b=5 \\ a^2+b^3=17 \end{cases} <=> \begin{cases} a=5-b \\ (5-b)^2+b^3=17 \end{cases}$ 1p

$b^2-10b+25+b^3-17=0 => b^3+b^2-10b+8=0$ 2p

$(b-2)(b^2+3b-4)=0$ 3p

I) $b=2\ ; a=3 <=> \sqrt{x+5}=3 <=> x+5=9 <=> x=4$ 1p

II) $b=1\ ; a=4 <=> \sqrt{x+5}=4 <=> x+5=16 <=> x=11$ 1p

III) $b=-4\ ; a=9 <=> \sqrt{x+5}=9 <=> x+5=81 <=> x=76$ 1p

b) $\bar{x}+\bar{y}+\bar{z}=0$ 1p

$x \cdot \bar{x} = |x|^2 = 1 => \bar{x} = \frac{1}{x}$ (1) 2p

$y \cdot \bar{y} = |y|^2 = 2 => \bar{y} = \frac{2}{y}$ (2) 2p

$z \cdot \bar{z} = |z|^2 = 3 => \bar{z} = \frac{3}{z}$ (3) 2p

Din (1) , (2) și (3)

$\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=0 <=> \frac{1}{x}+\frac{2}{y}-\frac{3}{x+y}=0$....................2p

$<=> 2x^2+y^2=0$ 1p

c) Grafic 1p

I) Dacă $m > 2 => f(2) > 4 => f$ nu este surjectivă 1p

II) Dacă $m < 2 => f(2) < 4 => f$ nu este injectivă 1p

f bijectivă $<=> m=2$ 1p

$f:\mathbb{R} \to \mathbb{R} = \begin{cases} -x^2+4x\ , x \in (-\infty, 2] \\ x+2\ , x \in (2, \infty) \end{cases}$ 1p

$f:(-\infty,2] \cup (2,\infty) \to (-\infty,4] \cup (4,\infty)$

$f^{-1}:(-\infty,4] \cup (4,\infty) \to \mathbb{R}$ 1p

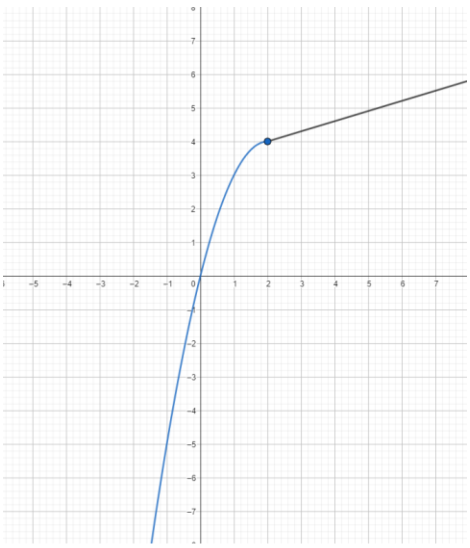
f surjectivă

$(\forall)\ y \in (4,\infty)\ (\exists) x \in (2,\infty)\ a.î.\ f(x)=y$

$x+2=y <=> x=y-2$ 1p

$(\forall)\ y \in (-\infty,4]\ (\exists) x \in (-\infty,2]\ a.î.\ f(x)=y$

$<=> x = 2-\sqrt{4-y} \in (-\infty,2]$ 2p



$f^{-1}(x)=\begin{cases}2-\sqrt{4-x}, x\in(-\infty,4]\\ x-2, x\in(4,\infty)\end{cases}$ 1p

Subiectul II (30p)

a) C.E: $x>0 => D=(0,\infty)$ 1p

$\log_6(\sqrt{x}+\sqrt[4]{x})=\frac{1}{2}\log_4 x$ 1p

$\log_6(\sqrt{x}+\sqrt[4]{x})=\log_4\sqrt{x}=t$ 1p

$\sqrt{x}=4^t => x=16^t$ 1p

$\log_6(\sqrt{x}+\sqrt[4]{x})=t <=> \sqrt{x}+\sqrt[4]{x}=6^t$ 1p

$\sqrt{16^t}+\sqrt[4]{16^t}=6^t <=> 4^t+2^t=6^t$ 1p

Ecuația $4^t+2^t=6^t$ 1p

$g(t)=\left(\frac{2}{3}\right)^t+\left(\frac{1}{3}\right)^t$ descrescătoare și injectivă 1p

$g(t)=1$ are cel mult o soluție 1p

$t=1 => x=16$ 1p

b) C.E: $x>0 => D=(0,\infty)$ 1p

$(\log_2 x)\cdot\left(\frac{\log_3 8x}{\log_3 81}\right)=\log_3 2 <=> \frac{(\log_2 x)(\log_3 8x)}{4}=\log_3 2$ 1p

Notăm $\log_3 x=t => t\cdot(3\log_3 2+t)=4\log_3^2 2$ 2p

$<=> t^2+3\log_3 2\cdot t-4\log_3^2 2=0$ 1p

$<=> (t+4\log_3 2)\cdot(t-\log_3 2)=0$ 3p

I) $t+4\log_3 2=0 <=> t=-4\log_3 2 <=> \log_3 x=\log_3\frac{1}{16} <=> x=\frac{1}{16}$ 1p

II) $t-\log_3 2=0 <=> t=\log_3 2 <=> \log_3 x=\log_3 2 <=> x=2$ 1p

c) $(m+3)\log_2^2 x-2m\log_2 x+m+5=0$ 1p

Notăm $\log_2 x=t\,; x>0\,; t\in\mathbb{R}$

$(m+3)t^2-2mt+(m+5)=0$ are două rădăcini reale $<=> \Delta>0$ 1p

$=> m<-\frac{15}{8}, m\in\left(-\infty,-\frac{15}{8}\right)$ 2p

$t_1+t_2=\frac{2m}{m+3} <=> \log_2 x_1+\log_2 x_2=\frac{2m}{m+3}$ 2p

$=> \log_2(x_1\cdot x_2)=\frac{2m}{m+3}$ 1p

$\frac{2m}{m+3}=\log_2 16=4$ 1p

$2m=4m+12 <=> m=-6\in\left(-\infty,-\frac{15}{8}\right)$ 2p

Subiectul III (30p)

a) $81^{\sin^2 x}+81^{1-\sin^2 x}=30$ 1p

$81^{\sin^2 x}+\frac{81}{81^{\sin^2 x}}=30$ 1p

Notăm $81^{\sin^2 x}=t>0$

$t_1=3\,, t_2=27$ 2p

MINISTERUL EDUCAȚIEI

I) $81^{\sin^2 x} = 3$ 1p

$\sin x = \frac{1}{2} \Rightarrow x = (-1)^k \cdot \arcsin\frac{1}{2} + k\pi\ ; k \in \mathbb{Z}$

$x = (-1)^k \cdot \frac{\pi}{6} + k\pi; k \in \mathbb{Z}$ 1p

$\sin x = -\frac{1}{2} \Rightarrow x = (-1)^k \cdot \arcsin(-\frac{1}{2}) + k\pi\ ; k \in \mathbb{Z}$

$x = (-1)^{k+1} \cdot \frac{\pi}{6} + k\pi; k \in \mathbb{Z}$ 1p

II) $81^{\sin^2 x} = 27$ 1p

$\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = (-1)^k \cdot \arcsin\frac{\sqrt{3}}{2} + k\pi\ ; k \in \mathbb{Z}$

$x = (-1)^k \cdot \frac{\pi}{3} + k\pi; k \in \mathbb{Z}$ 1p

$\sin x = -\frac{\sqrt{3}}{2} \Rightarrow x = (-1)^k \cdot \arcsin(-\frac{\sqrt{3}}{2}) + k\pi\ ; k \in \mathbb{Z}$

$x = (-1)^{k+1} \cdot \frac{\pi}{3} + k\pi; k \in \mathbb{Z}$ 1p

b) $D = \mathbb{R}\ ; \sqrt{(2+\sqrt{3})^x \cdot (2-\sqrt{3})^x} = 1$..........2p

Notăm $\sqrt{(2+\sqrt{3})^x} = t > 0 \Rightarrow t + \frac{1}{t} = 4 \Rightarrow t^2 - 4t + 1 = 0$..........2p

$\Rightarrow t_{1,2} = 2 \pm \sqrt{3}$..........2p

I) $\sqrt{(2+\sqrt{3})^x} = 2 + \sqrt{3} \Rightarrow x = 2$..........2p

II) $\sqrt{(2+\sqrt{3})^x} = 2 - \sqrt{3} = (2+\sqrt{3})^{-1} \Rightarrow x = -2$..........2p

c) $(|z_1| - |z_2|)^2 = |z_1|^2 - 2 \cdot |z_1| \cdot |z_2| + |z_2|^2 =$ 1p

$= a_1^2 + b_1^2 - 2\sqrt{a_1^2 + b_1^2} \cdot \sqrt{a_2^2 + b_2^2} + a_2^2 + b_2^2 \le (a_1^2 + b_1^2) + 2(a_1a_2 + b_1b_2) + (a_2^2 + b_2^2)$

$= (a_1 + a_2)^2 + (b_1 + b_2)^2 = |z_1 + z_2|^2 \Rightarrow \big||z_1| - |z_2|\big| \le |z_1 + z_2|$ (1) 4p

Avem : $|z_1 + z_2|^2 = (z_1 + z_2)(\overline{z_1} + \overline{z_2}) = |z_1|^2 + z_1\overline{z_2} + \overline{z_1}z_2 + |z_2|^2 =$ 2p

$= |z_1|^2 + |z_2|^2 + 2\mathrm{Re}(z_1 \cdot \overline{z_2}) \le (|z_1| + |z_2|)^2 \Rightarrow |z_1 + z_2| \le |z_1| + |z_2|$ (2) 2p

Din (1) și (2) $\Rightarrow \big||z_1| - |z_2|\big| \le |z_1 + z_2| \le |z_1| + |z_2|$ 1p