



MINISTERUL EDUCAȚIEI



CONCURSUL NAȚIONAL DE MATEMATICĂ  
„TEHNICI MATEMATICE”- editia a XIX-a  
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Clasa a X -a Matematică *M\_tehnologic*

Barem de corectare

Subiectul I (30p)

a)  $\sqrt{(\sqrt{x-2}-2)^2} + \sqrt{(\sqrt{x-2}+2)^2} = \sqrt{2024}$  .....2p

$|\sqrt{x-2}-2| + |\sqrt{x-2}+2| = \sqrt{2024}, x \in [2, \infty)$  .....2p

Notăm  $\sqrt{x-2} = t, t \geq 0$

$|t-2| + |t+2| = \sqrt{2024}$  .....1p

$|t-2| + t + 2 = \sqrt{2024}$  .....1p

I) Pt  $t \in [0,2]$  obținem  $4 = \sqrt{2024}$  (F) .....1p

II) Pt  $t \in (2, \infty)$ ,  $|t-2| = t-2$  .....1p

$\Rightarrow \sqrt{x-2} = \sqrt{506} \Leftrightarrow x-2 = 506 \Leftrightarrow x = 508$  .....2p

b) C.E. :  $\begin{cases} 10-x \geq 0 \\ 18-\sqrt{10-x} \geq 0 \end{cases} \Leftrightarrow x \in [-314,10] = D$  .....2p

$\sqrt{45}^{\sqrt{18-\sqrt{10-x}}} = (45)^2 \Leftrightarrow (45)^{\frac{1}{2}\sqrt{18-\sqrt{10-x}}} = 45^2$  .....2p

$\Leftrightarrow \sqrt{18-\sqrt{10-x}} = 4 \uparrow^2$  .....4p

Soluție  $x = 6$  .....2p

c) Notăm  $5^{3x} = t, t > 0$

$\sqrt{t+19} - \sqrt{t-4} = 1; t \in [4, \infty)$  .....2p

$\sqrt{t+19} = 1 + \sqrt{t-4} \uparrow^2 \Leftrightarrow \sqrt{t-4} = 11 \uparrow^2$  .....4p

$\Leftrightarrow t = 125 \in [4, \infty)$  .....2p

$5^{3x} = 125 \Leftrightarrow 5^{3x} = 5^3 \Leftrightarrow x = 1$  .....2p

Subiectul II (30p)

a)  $z^2 + z + 1 = 0$  (1) .....1p

$z^3 - 1 = (z-1)(z^2 + z + 1) = 0$  (2) .....2p

Din (1) și (2) avem  $z^3 = 1$  .....1p

$A = \frac{(1+z+z^2)+\dots+(z^{2022}+z^{2023}+z^{2024})+z^{2025}}{z^2+1}$  .....2p

$A = \frac{(1+z+z^2)+z^3(1+z+z^2)+\dots+z^{2022}(1+z+z^2)+z^{2025}}{z^2+1}$  .....2p

$A = \frac{z^{2025}}{z^2+1} = \frac{1}{z^2+1} = \frac{1}{-z} = -\frac{1}{z}$  .....2p



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- b)  $z_1 = \frac{2+4i}{3+i} = 1 + i$  .....2p  
 $(1 + i)^{2024} = 2^{1012} \cdot i^{1012} = 2^{1012}$  .....2p  
 $z_2 = \frac{1-3i}{2-i} = 1 - i$  .....2p  
 $(1 - i)^{2024} = (-2i)^{1012} = 2^{1012}$  .....2p  
 $\Rightarrow z_1^{2024} + z_2^{2024} = 2^{1012} + 2^{1012} = 2^{1013}$  .....2p  
c)  $P = [(a + ib) \cdot (a + i^2b) \cdot (a + i^3b) \cdot (a + i^4b)] \dots [(a + i^{2021}b) \cdot (a + i^{2022}b) \cdot (a + i^{2023}b) \cdot (a + i^{2024}b)]$  .....2p  
 $(a + ib) \cdot (a + i^2b) \cdot (a + i^3b) \cdot (a + i^4b) = (a + ib) \cdot (a - b) \cdot (a - ib) \cdot (a + b) = \dots$ 3p  
 $= (a + ib) \cdot (a - ib) \cdot (a + b) \cdot (a - b) = (a^2 + b^2) \cdot (a^2 - b^2) = a^4 - b^4$  .....4p  
 $\Rightarrow P = (a^4 - b^4)^{506}$  .....1p

**Subiectul III (30p)**

- a) C.E. :  $x > 0 \Rightarrow D = (0, \infty)$  .....1p  
 $4^{\log_9 x} - 6 \cdot 2^{\log_9 x} + 8 = 0$  .....1p  
Notăm  $\log_9 x = t$   
 $4^t - 6 \cdot 2^t + 8 = 0$  .....2p  
 $2^t = a > 0$  .....1p  
 $(a - 2)(a - 4) = 0$  .....3p  
I)  $a = 2 \Rightarrow x = 9$  .....1p  
II)  $a = 4 \Rightarrow x = 81$  .....1p

- b) C.E. :  $x > 0 \Rightarrow D = (0, \infty)$  .....1p  
 $(\log_3 x^2)^2 + 3 \cdot \log_3 x - 7 = 0$  .....1p  
 $(2 \log_3 x)^2 + 3 \cdot \log_3 x - 7 = 0$  .....1p  
 $4 \log_3^2 x + 3 \cdot \log_3 x - 7 = 0$  .....1p  
Notăm  $\log_3 x = t \Rightarrow 4t^2 + 3t - 7 = 0$  .....1p  
 $t_1 = 1, t_2 = -\frac{7}{4}$  .....2p  
I)  $\log_3 x = 1 \Rightarrow x = 3$  .....1p  
II)  $\log_3 x = -\frac{7}{4} \Rightarrow x = 3^{-\frac{7}{4}} \Rightarrow x = \frac{1}{3^{\frac{7}{4}}}$  .....2p

- c)  $A = Im f, y \in Im f$ ; ecuația:  $f(x) = y$  are cel puțin o soluție reală :  $\frac{x^2-4x+3}{x^2+x+1} = y$  .....1p  
 $\Leftrightarrow x^2 - 4x + 3 = yx^2 + yx + y \Leftrightarrow (y - 1)x^2 + (y + 4)x + (y - 3) = 0$  .....2p  
•  $y = 1 \Rightarrow 5x - 2 = 0 \Leftrightarrow x = \frac{2}{5} \Rightarrow 1 \in Im f$  .....1p  
•  $y \neq 1 \Rightarrow$  ecuația are cel puțin o soluție  $\Leftrightarrow \Delta \geq 0$   
 $\Leftrightarrow (y + 4)^2 - 4(y - 1)(y - 3) \geq 0$  .....2p  
 $\Leftrightarrow -3y^2 + 24y + 4 \geq 0 \Rightarrow y \in \left[ \frac{12-\sqrt{156}}{3}, \frac{12+\sqrt{156}}{3} \right] \Rightarrow y \in \left[ \frac{12-2\sqrt{39}}{3}, \frac{12+2\sqrt{39}}{3} \right]$  .....3p  
 $\Rightarrow A = Im f = \left[ \frac{12-2\sqrt{39}}{3}, \frac{12+2\sqrt{39}}{3} \right]$  (observăm că  $1 \in A$ ) .....1p