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Concursul Interjudețean „Mathematica – Modus Vivendi”
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BAREM CLASA a XI –a

1) det (A+xB)= det(B)x^3 + px^2 +qx + det (A)= f............................2p
f($\sqrt{2}$)= det (A + $\sqrt{2}\,B$) = det (A) +$\sqrt{2}$ det (B)
f(-$\sqrt{2}$) = det (A - $\sqrt{2}$ B) = det (A) - $\sqrt{2}$ det (B)..............................1p
2p+$\sqrt{2}$q+ $\sqrt{2}$det(B)=0,   2p-$\sqrt{2}$q- $\sqrt{2}$det(B)=0,
p=0, q= -det(B)..1p
f(-i)=det(A)+2idet(B), f(i)= det(A)-2idet(B)..................................1p
det (A^2 + B^2) =det(A-iB) det(A+iB)................ ....1p
det(A-iB)=f(-i), det(A+iB)=f(i)
det (A^2 + B^2) =$(\det(A))^2$+ $4(\det(B))^2$...1p

2) a) $4-x-3\sqrt[3]{x^2}=(\sqrt[3]{x}+2)^2\,(1-\sqrt[3]{x})$ ……………………………………2p
b) $x-3\sqrt[3]{x}+2=(\sqrt[3]{x}-1)^2\,(\sqrt[3]{x}+2)$……………………………………2p
Limita este $\left(\frac{3}{-0}\right)^{\frac{-1}{3}}=(-\infty)^{\frac{-1}{3}}=0$……………………………………3p

3) a) $x_2=\frac{7}{9}$, $x_3=\frac{89}{225}$……………………………………………………2p
b)Demonstrăm prin inductie că $x_{n+1}<1$……………………………1p
Din P(n) rezulta P(n+1) ⇔ $x_{n+1}<\frac{1}{2n+1}+\frac{2n+2}{(2n+1)^2}$ 1p
$x_{n+1}<\frac{4n+3}{4n^2+4n+1}<1$,P(n+1) adevărat ………………………………1p

$0<x_{n+1}<\frac{1}{2n+1}+\frac{2n+2}{(2n+1)^2}$ atunci $\lim_{n\to\infty} x_{n+1}=0$ 1p

$\lim_{n\to\infty} x_n=\lim_{n\to\infty}\frac{n x_{n-1}}{2n-1}+\frac{2n^2}{(2n-1)^2}=\frac{1}{2}$ ………………………………...1p

4) a) $Tr(A^*)=\frac{-1}{2}Tr(A^2)$………………………………………………2p
b) $A^3-Tr(A)A^2+Tr(A^*)A-\det(A)I_3=O_3$ (1)……………………1p
Aplicăm urma ecuației precedente
Obținem -24 – 3det(A)=0, det (A) = -8 ……………………………1p
Relația (1) devine $A^3+3A+8I_3=O_3$……………………………..1p
Det (A) diferit de zero exista inversa lui A…………………………1p
$A^{-1}=\frac{-1}{8}A^2-\frac{3}{8}I_3$ 1p