Colegiul Naţional „Mircea cel Bătrân”, Râmnicu-Vâlcea
Concursul Regional „Mathematica – Modus Vivendi”
Ediţia a XIX-a, 23 martie 2024
BAREM CLASA a IX -a

1. Din inegalitatea mediilor avem: $\frac{1}{x_2(x_1+x_2)}+\frac{1}{x_3(x_2+x_3)}+\ldots+\frac{1}{x_n(x_{n-1}+x_n)}+\frac{1}{x_1(x_n+x_1)}\geq$

$\geq \frac{n}{\sqrt[n]{x_1x_2\ldots x_n(x_1+x_2)(x_2+x_3)\ldots(x_{n-1}+x_n)(x_n+x_1)}}$ 3p

Dar $\sqrt[n]{x_1x_2\ldots x_n}\leq\frac{x_1+x_2+\ldots+x_n}{n}$ 1p

$\sqrt[n]{(x_1+x_2)(x_2+x_3)\ldots(x_{n-1}+x_n)(x_n+x_1)}\leq\frac{x_1+x_2+x_2+x_3+\ldots+x_{n-1}+x_n+x_n+x_1}{n}=$

$=\frac{2(x_1+x_2+\ldots+x_n)}{n}$ 2p

Finalizare 1p

Total = 7 puncte

2. **a.** $\{x\}\in[0,1), \{x\}\in\mathbb{Z}\Rightarrow\{x\}=0\Rightarrow\left[\frac{|x+1|}{2}\right]=0$..1p

$\Rightarrow 0\leq\frac{|x+1|}{2}<1\Rightarrow 0\leq|x+1|<2\Rightarrow -2<x+1<2\Rightarrow x\in(-3,1)$..1p

Cum $\{x\}\in\mathbb{Z}\Rightarrow x\in\{-1,0\}$..1p

b.Notăm $x+\frac{3}{2}=y\Rightarrow x=y-\frac{3}{2}$

Ecuația devine $[y]+\left[y+\frac{1}{2}\right]-[4y]=-\frac{2y}{3}$..1p

$[2y]-[4y]=-\frac{2y}{3}\Rightarrow\left[2y+\frac{1}{2}\right]=\frac{2y}{3}$ (am folosit relația Hermite) 1p

Folosind notația $\frac{2y}{3}=k\in\mathbb{Z}$ avem $k\leq 3k+\frac{1}{2}<k+1\Rightarrow 0\leq 2k+\frac{1}{2}<1$ 1p

Obținem $k=0\Rightarrow x=-\frac{3}{2}$. 1p

Total = 7 puncte

3. **a.** $\sum_{k=1}^{10}b_kb_{10-k+1}=512\Rightarrow 10b_1b_{10}=512$ 1p

$\Rightarrow 10b_1^2q^9=512\Rightarrow q^9=512\Rightarrow q=2$ 1p

b. $S_{2n}=b_1b_{2n}\left(\frac{b_1+b_{2n}}{b_1b_{2n}}+\frac{b_2+b_{2n-1}}{b_2b_{2n-1}}+\ldots+\frac{b_n+b_{n+1}}{b_nb_{n+1}}\right)=b_1b_{2n}\left(\frac{1}{b_1}+\frac{1}{b_2}+\ldots+\frac{1}{b_{2n}}\right)=$……………..2p

$b_1b_{2n}\frac{1}{S_{2n}}\left(b_1+b_2+\ldots+b_{2n}\right)\left(\frac{1}{b_1}+\frac{1}{b_2}+\ldots+\frac{1}{b_{2n}}\right)\geq\frac{4n^2b_1b_{2n}}{S_{2n}}\Rightarrow b_1b_{2n}\leq\left(\frac{S_{2n}}{2n}\right)^2$ …………………2p

Am ținut seama de inegalitatea dintre media armonică și media aritmetică

$\frac{2n}{\frac{1}{b_1}+\frac{1}{b_2}+\ldots+\frac{1}{b_{2n}}}\leq\frac{b_1+b_2+\ldots+b_{2n}}{2n}\Rightarrow\left(b_1+b_2+\ldots+b_{2n}\right)\left(\frac{1}{b_1}+\frac{1}{b_2}+\ldots+\frac{1}{b_{2n}}\right)\geq 4n^2$ …………1p

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Total = 7 puncte

4. **a.** Rezultă imediat că $\frac{C'A}{C'B}=\frac{1}{k},\frac{B'A}{B'C}=\frac{k-1}{k}$ ………………………………………………..1p

Aplicăm teorema Ceva și obținem: $\frac{1}{k}\cdot\frac{\overrightarrow{A'B}}{\overrightarrow{A'C}}\cdot\frac{k}{k-1}=-1\Rightarrow\overrightarrow{BA'}=(k-1)\overrightarrow{A'C}$……………..1p

b. Avem $\overrightarrow{MA'}=\frac{\overrightarrow{MB}+(k-1)\overrightarrow{MC}}{k},\ \overrightarrow{AA'}=\frac{\overrightarrow{AB}+(k-1)\overrightarrow{AC}}{k}$……………………………..………..2p

Aplicând teorema Menelaus în $\triangle AA'C$ cu transversala $\overline{B'MB}$ obținem:

$\frac{\overrightarrow{B'A}}{\overrightarrow{B'C}}\cdot\frac{\overrightarrow{BC}}{\overrightarrow{BA'}}\cdot\frac{\overrightarrow{MA'}}{\overrightarrow{MA}}=1$…………………………………………………………………......1p

$\Rightarrow-\frac{k-1}{k}\cdot\frac{k}{k-1}\cdot\frac{\overrightarrow{MA'}}{\overrightarrow{MA}}=1\Rightarrow\overrightarrow{MA'}=\overrightarrow{AM}\Rightarrow\overrightarrow{AA'}=2\overrightarrow{MA'}$………………………………………..1p

$\frac{\overrightarrow{AB}+(k-1)\overrightarrow{AC}}{k}=2\frac{\overrightarrow{MB}+(k-1)\overrightarrow{MC}}{k}\Rightarrow$

$\Rightarrow\overrightarrow{AB}-2\overrightarrow{MB}=(1-k)\left(\overrightarrow{AC}-2\overrightarrow{MC}\right)$..1p

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Total = 7 puncte